Euro-Maghreb Conference in Algebra,
Geometry and Lie Theory
Sousse, April 25-28, 2023

Structure of BiHom-Poisson algebras and ternary BiHom-Poisson algebras

*** Amri Hanene[1], Sami Mabrouk[2], Hadjer Adimi[3], Abdenacer Makhlouf[4]**

*** University of Annaba[1], University of Gafsa[2], University of Bordj Bou Arréridj [3], University of Haute Alsace[4]**

Abstract:
A vector space A is called a Poisson algebra provided that, beside addition, it has two $\mathbb{K}$-bilinear operations which are related by derivation. First, with respect to multiplication, A is a commutative associative algebra; denote the multiplication by $\mu(a,b)$ (or $a \cdot b$ or ab), where $a, b \in A$. Second, A is a Lie algebra; traditionally here the Lie operation is denoted by the Poisson brackets {a,b}, where $a, b \in A$. It is also assumed that these two operations are connected by the Leibniz rule

$$\{a \cdot b, c\}= a \cdot \{b, c\} + b \cdot \{a, c\},\ a,\ b,\ c \in A\ [4,\ 8].$$

Poisson algebras are the key to recover Hamiltonian mechanics and are also central in the study of quantum groups. Manifolds with a Poisson algebra structure are known as Poisson manifolds, of which the symplectic manifolds and the Poisson-Lie groups are a special case. Their generalization is known as Nambu algebras [9, 3, 1, 2], where the binary bracket is generalized to ternary or n-ary bracket.

Motivated by a categorical study of Hom-algebras and new type of categories, generalized algebraic structures endowed with two commuting multiplicative linear maps, called BiHom-algebras including BiHom-associative algebras, BiHom-Lie algebras and BiHom-Bialgebras were introduced in [5]. Therefore, when the two linear maps are the same, BiHom-algebras will be turn to Hom-algebras in some cases. Various studies deal with these new type of algebras, see [11, 6, 7] and references therein.

The aim of this talk is to study BiHom-Poisson algebras, in particular Non-BiHom-Commutative BiHom-Poisson algebras. We discuss their representation theory and Semi-direct product. Furthermore, we characterize admissible BiHom-Poisson algebras, and we establish the classification of 2-dimensional BiHom-Poisson algebras.

Then from BiHom-Poisson algebras, the Ternary BiHom-Poisson algebras is constructed using generalized trace function, they are called Ternary BiHom-Poisson algebras induced by BiHom-Poisson algebras and provide example of ternary BiHom-Poisson algebra obtained using this construction.

Keywords: BiHom-algebra, BiHom-Poisson algebra, Ternary BiHom-Poisson algebra.
2010 Mathematics Subject Classification: 17A32,17A40,17B61, 17B63.

References

[1] G. Dito, M. Flato, D. Sternheimer, and L. Takhtajan, Deformation quantization and Nambu mechanics. Commun. Math. Phys., **183**, 1–22, (1997).

[2] G. Dito, M. Flato, and D. Sternheimer. Nambu mechanics, n-ary operations and their quantization.In Deformation Theory and Symplectic Geometry. Math. Phys. Stud. **20**, Kluwer Academic Publishers, Dordrecht, Boston, 43–66, (1997).

[3] V.T. Filippov, n-Lie algebras,(Russian), Sibirsk. Mat. Zh. **26**, no. 6, 126–140 (1985). (English translation: Siberian Math. J. **26**, no. 6, 879–891, (1985))

[4] M. Goze and E. Remm, Poisson algebras in terms of non-associative algebras, J. Alg. 320 (2008) 294–317.

[5] G. Graziani, A. Makhlouf, C. Menini, F. Panaite, BiHom-associative algebras, BiHom-Lie algebras and BiHom-bialgebras. Symmetry, Integrability and Geometry : Methods and Applications SIGMA 11 (2015), 086, 34 pages

[6] L. Liu, A. Makhlouf, C. Menini, F. Panaite, BiHom-pre-Lie algebras, BiHom-Leibniz algebras and Rota-Baxter operators on BiHom-Lie algebras, arXiv:1706.00474. (2017).

[7] L. Liu, A. Makhlouf, C. Menini, F. Panaite, Rota-Baxter operators on BiHom-associative algebras and related structures. Colloq. Math., Vol. 161 (2020), 263–294.

[8] F. Kubo, Finite-dimensional non-commutative Poisson algebras, J. Pure Appl. Alg. 113 (1996) 307–314.

[9] Y. Nambu, Generalized Hamiltonian mechanics, Phys. Rev. D (3), **7**, 2405–2412, (1973).

[10] D. Yau, Non-commutative Hom-Poisson algebras, arXiv:1010.3408 v1 [math.RA], (2010).

[11] C. Yongsheng, and H. Qi, Representations of Bihom-Lie algebras. arXiv:1610.04302 (2016).