

ON POLYNOMIAL CONJECTURES OF NILPOTENT LIE GROUPS UNITARY REPRESENTATIONS

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Abstract

Let G be a connected and simply connected nilpotent Lie group of Lie algebra \mathfrak{g} , K an analytic subgroup of G , χ a unitary character of G and π an irreducible unitary representation of G . In this setting, the orbit method allows to identify the unitary dual of G to the space of coadjoint orbits. Using the enveloping algebra of the complexified Lie algebra of \mathfrak{g} , we consider two algebras of differential operators $D\pi(G)^K$ and $D\tau(G/K)$ associated respectively to the restriction $\pi|_K$ of π to K and to the monomial representation $\tau = \text{Ind}_G^K \chi$. Under the assumption that these representations are of finite multiplicities, the polynomial conjectures stating that $D\pi(G)^K$ and $D\tau(G/K)$ are K -invariant polynomial rings hold. In this lecture, I will overview some history of the conjectures and some restrictive cases. Once restricted to codimension one normal subgroups of G , the study of the geometry and the saturation of coadjoint orbits plays a crucial role in the proofs.

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