On gradations of Kac-Moody Lie algebras by Kac-Moody root systems

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Abstract

The notion of gradation of a Lie algebra $\mathfrak{g}$ by a finite root system Σ was introduced by S. Berman and R. Moody in 1992. Since then, many authors have been interested in the subject to give a complete classification (up to isogeny) of perfect Lie algebras graded by finite root systems (G. Benkart, E. Zelmanov, E. Neher, B. Allison, Y. Gao, J. Nervi, ...). This notion was extended by J. Nervi (2000) to the case where $\mathfrak{g}$ is an affine Kac-Moody algebra and Σ the (infinite) root system of an affine Kac-Moody subalgebra.

In this talk, I will first recall the construction of Kac-Moody Lie algebras as generalisation of semi-simple Lie algebras and then present a joint work with G. Rousseau on gradations of Kac-Moody Lie algebras by Kac-Moody root systems with finite dimensional weight spaces. We extend, to general Kac-Moody Lie algebras, the notion of $C-$admissible pairs as introduced by H. Rubenthaler and J. Nervi for semi-simple and affine Lie algebras. If $\mathfrak{g}$ is a Kac-Moody Lie algebra (with Dynkin diagram indexed by I) and (I, J) is such a $C-$admissible pair, we construct a $C-$admissible subalgebra $\mathfrak{g}^J$, which is a Kac-Moody Lie algebra of the same type as $\mathfrak{g}$, and whose root system Σ grades finitely the Lie algebra $\mathfrak{g}$. For an admissible quotient $\rho : I \to \overline{I}$ we build also a Kac-Moody subalgebra $\mathfrak{g}^\rho$ which grades finitely the Lie algebra $\mathfrak{g}$. If $\mathfrak{g}$ is affine or hyperbolic, we prove that the classification of the (real) gradations of $\mathfrak{g}$ is equivalent to those of the $C-$admissible pairs and of the admissible quotients. For general Kac-Moody Lie algebras of indefinite type, the situation may be more complicated and brings out the notion of imaginary graduation which will be discussed by M. Layouni in her talk.