Euro-Maghreb Conference in Algebra, Geometry and Lie Theory

Sousse, April 25-27, 2023

Royal Kenz Hôtel Thalasso & Spa

The aim of this conference is to gather experts and young researchers working in various area of algebra. It will be focused on nonassociative algebras, Lie theory and Deformation Theory. Recent trends and applications will be discussed.

Organizers : B. Agrebaoui, W. Aloulou, S. Mabrouk, A. Makhlouf, N. Ouled Aziz

Abdenacer Makhlouf
University Haute-Alsace, IRIMAS, Mulhouse, ,France

Curved $\mathcal{O}$-operators systems

Abstract

In this talk, we discuss the notion of curved $\mathcal{O}$-operator systems and then investigate the re- lations among $\mathcal{O}$-operator systems, (tri)dendriform systems, pre-Lie algebras, associative Yang- Baxter pairs and quasitriangular covariant bialgebras. This is a joint work with Tianshui Ma and Sergei Silvestrov.

Quentin Ehret
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Symplectic double extensions for restricted quasi-Frobenius Lie (super)algebras

Abstract

This talk will be devoted to introduce a method for symplectic double extensions of restricted quasi-Frobenius Lie superalgebras. Certain cocycles of the restricted cohomology represent obstructions to symplectic double extension, which will be fully described. Then, we will present necessary conditions for a restricted quasi-Frobenius Lie superalgebra to be a symplectic double extension of a smaller restricted Lie superalgebra. Finally, we will illustrate these constructions with a few examples.

This is a joint work with Sofiane Bouarroudj and Yoshiaki Maeda.

Hechmi Ben Messaoud

On gradations of Kac-Moody Lie algebras by Kac-Moody root systems

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Abstract

The notion of gradation of a Lie algebra $\mathfrak{g}$ by a finite root system Σ was introduced by S. Berman and R. Moody in 1992. Since then, many authors have been interested in the subject to give a complete classification (up to isogeny) of perfect Lie algebras graded by finite root systems (G. Benkart, E. Zelmanov, E. Neher, B. Allison, Y. Gao, J. Nervi, ...). This notion was extended by J. Nervi (2000) to the case where $\mathfrak{g}$ is an affine Kac-Moody algebra and Σ the (infinite) root system of an affine Kac-Moody subalgebra.

In this talk, I will first recall the construction of Kac-Moody Lie algebras as generalisation of semi-simple Lie algebras and then present a joint work with G. Rousseau on gradations of Kac-Moody Lie algebras by Kac-Moody root systems with finite dimensional weight spaces. We extend, to general Kac-Moody Lie algebras, the notion of $C-$admissible pairs as introduced by H. Rubenthaler and J. Nervi for semi-simple and affine Lie algebras. If $\mathfrak{g}$ is a Kac-Moody Lie algebra (with Dynkin diagram indexed by I) and (I, J) is such a $C-$admissible pair, we construct a $C-$admissible subalgebra $\mathfrak{g}^J$, which is a Kac-Moody Lie algebra

of the same type as $\mathfrak{g}$, and whose root system Σ grades finitely the Lie algebra $\mathfrak{g}$. For an admissible quotient $\rho : I \to \overline{I}$ we build also a Kac-Moody subalgebra $\mathfrak{g}^\rho$ which grades finitely the Lie algebra $\mathfrak{g}$. If $\mathfrak{g}$ is affine or hyperbolic, we prove that the classification of the (real) gradations of $\mathfrak{g}$ is equivalent to those of the $C-$admissible pairs and of the admissible quotients. For general Kac-Moody Lie algebras of indefinite type, the situation may be more complicated and brings out the notion of imaginary graduation which will be discussed by M. Layouni in her talk.

ON POLYNOMIAL CONJECTURES OF NILPOTENT LIE GROUPS UNITARY REPRESENTATIONS

ALI BAKLOUTI
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Abstract

Let G be a connected and simply connected nilpotent Lie group of Lie algebra $\mathfrak{g}$, K an analytic subgroup of G, χ a unitary character of G and π an irreducible unitary representation of G. In this setting, the orbit method allows to identify the unitary dual of G to the space of coadjoint orbits. Using the enveloping algebra of the complexified Lie algebra of $\mathfrak{g}$, we consider two algebras of differential operators $D\pi(G)^K$ and $D\tau(G/K)$ associated respectively to the restriction $\pi|_K$ of π to K and to the monomial representation $\tau = Ind_G^K \chi$. Under the assumption that these representations are of finite multiplicities, the polynomial conjectures stating that $D\pi(G)$ K and $D\tau(G/K)$ are K-invariant polynomial rings hold. In this lecture, I will overview some history of the conjectures and some restrictive cases. Once restricted to codimension one normal subgroups of G, the study of the geometry and the saturation of coadjoint orbits plays a crucial role in the proofs.

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Hauteur du joint de deux variétés projectives.

Zoheir Chebel
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Sylow's Theorems for Hom-groups

Abstract

The notion of hom-groups is defined as a generalization of a non-associative group. They can be obtained by twisting the associative operation with a compatible bijection mapping. In this, presentation, we give some definitions and

properties related to Hom-groups. We introduce the different notions of actions concerning a Hom-groups. At the end, we give the different Sylow's Theorems for the Hom group with their proofs.

Sami Ben Abdelhafidh
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Deformations and their controlling cohomologies of nonabelian embedding tensors on 3-Lie algebras

Abstract

In this paper, first we introduce the notion of a nonabelian embedding tensor on a 3-Lie algebra. In accordance with the general principles of deformation theories, a deformation theory of nonabelian embedding tensors is established. On the one hand, using the higher derived brackets, we construct an L∞-algebra whose Maurer-Cartan elements are nonabelian embedding tensors. Consequently, given a nonabelian embedding tensors T on a 3-Lie algebras, we obtain the twisted L∞-algebra that controls deformations of T. On the other hand, a 3-Lie algebra with a coherent action is identified from a nonabelian embedding tensors T such that the corresponding Loday-Pirashvili cohomology controls deformations of T. Finally, a linear deformations of nonabelian embedding tensors are studied. In particular, we introduce the notion of Nijenhuis elements on a 3-Lie algebras to characterize trivial linear deformations."

Mohamed Amin Sadraoui
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On Lie and pre-Lie Coder pairs

Abstract

On introduit le concept du Lie et pre-Lie Der pair ainsi la representation du Lie Coder pair et étudié les différentes relations avec quelconque operateurs algébrique.

Atef Hajjaji
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Cohomologies and deformations of twisted $\mathcal{O}$-operatorson 3-Lie algebras

Abstract

We introduce and study twisted $\mathcal{O}$-operators on 3-Lie algebras. We construct an L_∞-algebra whose Maurer-Cartan elements are twisted $\mathcal{O}$-operators and define a cohomology of a twisted $\mathcal{O}$-operator T as the Chevalley-Eilenberg cohomology of a certain 3-Lie algebra induced by T with coefficients in a suitable representation. Moreover we consider infinitesimal and formal deformations of twisted $\mathcal{O}$-operators.

Rahma Gharbi
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Maurer-Cartan type cohomology on generalized Reynolds operators and NS-structures on Lie triple systems.

Abstract

We introduce the notion of generalized Reynolds operators on Lie triple systems as particular case of weighted Reynolds operator on Lie triple systems and we construct an L_∞-algebra whose Maurer-Cartan elements are generalized Reynolds operators. By this fact, we define the Yamaguti cohomology of a generalized Reynolds operator. Moreover, we study deformations of generalized Reynolds operators from cohomological points of view and we investigate the obstruction class of an extendable deformation of order n. We end this paper by introducing a new algebraic structure, called NS-Lie triple system using a generalized Reynolds operator and we show that NS-Lie triple system can be constructed by NS-Lie algebra.

Fatma Zaouidi
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Lie-Poisson triple systems and related structures.

Abstract

We introduce the notion of pre-Lie Poisson triple systems. Pre-Lie Poisson triple systems are regarded as the underlying algebraic structures of Lie Poisson triple systems with nondegenerate symplectic form. They are also the algebraic structures behind the Rota-Baxter operators of Lie Poisson triple systems.

Kais Smaoui
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Hardy Uncertainty Principle for Gabor Transform on Connected Nilpotent Lie Group.

Abstract

We study the conjecture of Bansal, Kumar and Sharma, which is an analog of Hardy's theorem for Gabor transform in the setup of connected nilpotent Lie groups. To approach this conjecture, we use the orbit method and the Plancherel theory. When the Lie group G is simply connected, we show that the conjecture is true. Moreover, we extend this result further to connected nilpotent Lie groups with non-compact center.

Walid Mhiri
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The solenoidal-Virasoro algebra and its Harish-Chandra modules.

Abstract

Let $\mathbf{A} = \mathbb{C}[t_i, t_i^{-1},\ 1 \leq i \leq n]$ be the algebra of Laurent polynomials in n-variables and let $\mathbf{W}_\mu = \mathbf{A}d_\mu$ be the solenoidal Lie algebra introduced by Y.

Billig and V. Futorny in [**?**], where $\mu = (\mu_1, \ldots, \mu_n) \in \mathbb{C}^n$ is generic and

$$d_\mu = \sum_{i=1}^{n} \mu_i t_i \frac{d}{dt_i}.$$

In this paper, we prove the existence and the uniqueness of universal central extension of $\mathbf{W}_\mu$. Such extension will be called the solenoidal Virasoro algebra and will be denoted by $\mathbf{Vir}_\mu$. Then we study the Harish-Chandra modules of $\mathbf{Vir}_\mu$. We establish three classes of Harish-Chandra modules: the highest weight modules, the lowest weight modules and the cuspidal modules. In the case $n = 1$, our results coincide with the well known V. Kac conjecture (see [**?**, **?**]) wich is proved by O. Mathieu in [**?**].

Boujemaa Agrebaoui
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The solenoidal Heisenberg-Virasoro Lie algebra and its Harish-Chandra modules.

Abstract

Let $\mathbf{A} = \mathbb{C}[t_i, t_i^{-1},\ 1 \leq i \leq n]$ and $\mathbf{W}_\mu = \mathbf{A}d_\mu$ the solenoidal Lie algebra introduced in [**?**], where $\mu = (\mu_1, \ldots, \mu_n) \in \mathbb{C}^n$ is generic and $d_\mu = \sum_{i=1}^{n} \mu_i \frac{d}{dt_i}$. We consider the semi-direct product Lie algebra $\mathbf{WA}_\mu := \mathbf{W}_\mu \times \mathbf{A}$. We prove existence and unicity of the universal central extension of $\mathbf{WA}_\mu$ and we call it the solenoidal Virasoro algebra $\mathbf{HVir}_\mu$. Then we study Harish-Chandra modules of $\mathbf{HVir}_\mu$ and we establish that we have three classes: highest weight modules, lowest weight modules and cuspidal modules. In $n = 1$, our results coincide with R. Lü and K. Zhao [**?**]. We provide a theorem generalizing Kac conjecture [**?**, **?**] proved by Mathieu [**?**].

Jaber Faidi
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Realization of representations of the Hom-Lie algebra $\mathfrak{sl}_q(2, \mathbb{C})$ of Jackson.

Abstract

The weight modules of the Lie algebra $\mathfrak{sl}(2, \mathbb{C})$ are well known(for self contained reference see [**?**]). In the first part of this paper we deal with a realization of weight modules of $\mathfrak{sl}(2, \mathbb{C})$ in the space $t^\nu \mathbb{C}[t, t^{-1}],\ \nu \in \mathbb{R}$, where $\mathbb{C}[t, t^{-1}]$ is the algebra of Laurent polynomials.

In the second part, we consider the Hom-Lie algebra $\mathfrak{sl}_q(2, \mathbb{C})$ of Jackson where $q \neq 0, 1$. The q-analogue of the above realization in the space $t^\nu \mathbb{C}(q)[t, t^{-1}],\ \nu \in \mathbb{R}$ are considered. We obtain two kinds of q-modules. The regular q-modules which have limits the modules obtained in the classical realization when q go to 1 . The other q-modules have no limits when q go to 1 and they are called singular modules.

Hanene Amri
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Structure of BiHom-Poisson algebras and ternary BiHom-Poisson algebras.

Abstract

A vector space A is called a Poisson algebra provided that, beside addition, it has two K-bilinear operations which are related by derivation. First, with respect to multiplication, A is a commutative associative algebra; denote the multiplication by $\mu(a, b)$ (or $a.b$ or ab), where $a, b \in A$. Second, A is a Lie algebra, traditionally here the Lie operation is denoted by the Poisson brackets $\{a, b\}$, where $a, b \in A$. It is also assumed that these two operations are connected by the Leibniz rule $\{a \cdot b, c\} = a \cdot \{b, c\} + b \cdot \{a, c\}, a, b, c \in A$[4, 8]. Poisson algebras are the key to recover Hamiltonian mechanics and are also central in the study of quantum groups. Manifolds with a Poisson algebra structure are known as Poisson manifolds, of which the symplectic manifolds and the Poisson-Lie groups are a special case. Their generalization is known as Nambu algebras [9, 3, 1, 2], where the binary bracket is generalized to ternary or n-ary bracket. Motivated by a categorical study of Hom-algebras and new type of categories, generalized algebraic structures endowed with two commuting multiplicative linear maps, called BiHom-algebras including BiHom-associative algebras, BiHom-Lie algebras and BiHom-Bialgebras were introduced in [5]. Therefore, when the two linear maps are the same, BiHom-algebras will be turn to Hom-algebras in some cases. Various studies deal with these new type of algebras, see [11, 6, 7] and references therein. The aim of this talk is to study BiHom-Poisson algebras, in particular Non-BiHom-Commutative BiHom-Poisson algebras. We discuss their representation theory and Semi-direct product. Furthermore, we characterize admissible BiHom-Poisson algebras, and we establish the classification of 2-dimensional BiHom-Poisson algebras. Then from BiHom-Poisson algebras, the Ternary BiHom-Poisson algebras is constructed using generalized trace function, they are called Ternary BiHom-Poisson algebras induced by BiHom-Poisson algebras and provide example of ternary BiHom-Poisson algebra obtained using this construction.

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On the second $\mathfrak{osp}(1|2)$-relative cohomology of the Lie superalgebra of contact vector fields on $\mathcal{C}^{1|1}$.

Abstract

Let $\mathcal{K}(1)$ be the Lie superalgebra of contact vector fields on the (1,1)-dimensional complex superspace; it contains the Möbius superalgebra $\mathfrak{osp}(1|2)$. We classify $\mathfrak{osp}(1|2)$-invariant superanti-symmetric binary differential operators from $\mathcal{K}(1) \wedge \mathcal{K}(1)$ to $\mathfrak{D}_{\lambda,\mu}$ vanishing on $\mathfrak{osp}(1|2)$, where $\mathfrak{D}_{\lambda,\mu}$ is the superspace of linear differential operators acting on the superspaces of weighted densities. This result allows us to compute the second differential $\mathfrak{osp}(1|2)$-relative cohomology of $\mathcal{K}(1)$ with coefficients in $\mathfrak{D}_{\lambda,\mu}$.

Karima Benali
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Compatible Hom-mock-Lie algebras.

Abstract

we study Hom-mock-Lie algebras as a twisted version of mock-Lie algebras. Also we consider a pair of Hom-mock-Lie algebras structures satisfying that any linear combination of the two Hom-mock-Lie structures is still a Hom-mock-Lie structure called compatible Hom-mock-Lie algebras and exhibit some related results. Next, we introduce the notion of representation of a compatible Hom-mock-Lie algebra and give some related results.

Salah Benyoucef
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Hyperbolic algebraic limit cycle and Lie symmetry.

Abstract

We consider systems of differential equations of the form

$$\dot{x} = \frac{dx}{dt} = P(x, y),\ \dot{y} = \frac{dy}{dt} = Q(x, y).$$

where $P(x, y); Q(x, y)$ are real polynomials of the variables x, y, and $t \in \mathbb{R}$ is taken as an independent variable. The degree of the system is the maximum of the degrees of the polynomials P and Q. These differential systems are mathematical models and arise in great variety of applications, for example, ecology and population dynamics, chemical reaction and plasma physics etc. One of the most important topics in qualitative theory of planar dynamical systems is related to the second part of the unsolved Hilbert 16th problem which consisted to study the maximum number of limit cycles and their relative distributions in the plane of the real polynomial system of degree n. In this presentation we will study the existence of hyperbolic algebraic limit cycles for classes of polynomial differential systems in plane via vector field which generate Lie symmetry. Concrete examples exhibiting the applicability of result are introduced.

Key Words: Sixteenth problem of Hilbert, planar differential system, Invariant curve, Periodic solution, hyperbolic limit cycle, Lie symmetry.

2020 Mathematics Subject Classification: 34C05,34C07,34C2

Sami Mabrouk
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On (Bi)Hom-algebras and ternary Hom-algebras structures.
Abstract

We recall some basic on binary Bihom-algebras and 3-Hom-Lie algebras. Also, we introduce and study BiHom-alternative algebras and BiHom-Malcev algebras. It is shown that BiHom-alternative algebras are admissible BiHom-Malcev and BiHom-Jordan eligible. Moreover, we introduce the notion of Yau-type 3-Hom-Lie bialgebras and give some constructions.

Rafik Khalfi
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Benson-Ratcliff Conjecture on Solvable Lie algebras.
Abstract

In this paper, we study the conjecture of Benson and Ratcliff, which deals with the class of nilpotent Lie algebras of a one-dimensional center. We show that this conjecture is true for any nilpotent Lie algebra $\mathfrak{g}$ with $dim\mathfrak{g} \leq 5$, but it fails for the dimensions greater or equal to 6. To this end, we produce counter-examples to the Benson-Ratcliff conjecture in all dimensions $n \geq 6$. Finally, we show that this conjecture is true for the class of three-step nilpotent Lie algebras and for some other classes of nilpotent Lie algebras.

Khouloud Abid Baklouti
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Uncertainty Principle for Gabor Transform on nilpotent Lie Groups.
Abstract

We define and prove an analog of Heisenberg uncertainty inequality for Gabor transform in the setup of connected, simply connected nilpotent Lie groups. When G is connected nilpotent and has a non-compact center, a proof of such an analog is given for functions in the Schwartz space of G. The representation theory and a localized Plancherel formula are fundamental tools in the proof of our results.

Marwa Layouni
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Autour des graduations des algèbres de Kac-Moody par des systèmes de racines.
Abstract

Walid Aloulou
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Hom-pre-Poisson algebras and Hom-Gerstenhaber algebras up to homotopy.
Abstract

In this research, we study the concept of algebra up to homotopy for a structure defined by two operations. As important examples of this structure ; the

Gerstenhaber algebra (commutatitve and Lie) and pre-Poisson algebra (Zinbiel and pre-Lie). A Hom-Gerstenhaber algebra is defined by a structure of commutative and Hom-associative algebra and a structure of a Hom-Lie algebra. We will give an explicit construction of the associated Hom-Gerstenhaber algebra up to homotopy, this is a bicoalgebra (Hom-coLie and Hom-coassociative) equipped with a codifferential which is a coderivation for the two coproducts allowing the construction of HomG∞-algebra. Furthermore, if we have a structure of Hom-pre-Lie algebra and Hom-Zinbiel algebras, we define the structure of Hom-pre-Poisson algebras verifying two compatibility conditions. We demonstrate that when A is a Hom-pre-Lie algebra, then a tensoriel algebra of A has a structure of Hom-pre-Poisson algebra. On the other hand, we prove that any Hom-Poisson algebra equipped with a Baxter operator can define a structure of Hom-pre-Poisson algebra.

Hom-actions for Hom-groups

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The notion of the Hom-groups is defined as a generalization of a non-associative group. They can be obtained by twisting the associative operation with a compatible bijection mapping. In this work, we give some constructions by twisting and also give properties related to Hom-groups. We introduce the different notions of actions concerning a Hom-groups. Present the theorem for a class equation. There follows some application for p-Hom-groups are illustrated.

Key Words: Hom-groups; Hom-subgroups; Hom-quotient groups; Isomorphism; Hom-group actions; Class equation.
2010 Mathematics Subject Classification: 17A99, 17B61, 20B99, 20D20, 20N05.

Time	Tuesday 25th April	Wednesday 26th April	Thursday 27th April
8h30 - 9h10	Ouverture	Abdenacer Makhlouf	Boujemaa Agrebaoui
9h20 - 10h00	Ali Baklouti	Quentin Ehret	Nizar Ben Fraj
10h00 - 10h30	**Coffee break**	**Coffee break**	**Coffee break**
10h30 - 11h10	Walid Aloulou	Hechmi Ben Messaoud	Kais Smaoui
11h20 - 12h00	Zoheir Chebel	Najib Ouled Azeiz	Sami Mabrouk
12h00 - 14h00	**Lunch**	**Lunch**	**Lunch**
14h00 - 14h30	Sami Ben Abdelhafidh	Hanene Amri	
14h30 - 15h00	Mohamed Amin Sadraoui	Walid Mhiri	
15h00 - 15h30	Atef Hajjaji	Karima Benali	
15h30 - 16h00	Rahma Gharbi	Salah Benyoucef	
16h00 - 16h30	**Coffee break**	**Coffee break**	
16h30- 17h00	Fatma Zaouidi	Rafik Khalfi	
17h-17h30	Jaber Faidi	Marwa Layouni	
17h30- 18h00	Hadjer Adimi	Khouloud Abid	
18h00- 18h30			