Transport type metrics on the space of probability measures involving singular base measures.

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Abstract: In this talk we introduce a new metric based on a slight refinement of the notion of generalized geodesics with respect to a base measure ν, relevant in particular for the case when ν is singular with respect to $m-$ dimensional Lebesgue measure. Defined in terms of an interated variational problem involving optimal transport to ν, we also characterize the ν-based Wasserstein metric in terms of integrations of classical Waserstein distance between the conditional probabilities when measures are disintegrated with respect to optimal transport to ν, and through limits of certain multi-marginal optimal transport problems.

We establish geodesic convexity of the usual class of functionals and prove convergence of an iterative scheme to solve a variational problem arising in game theory.

Moreover we also use the multi-marginal formulation to characterize solutions to the multi-marginal problem by an ordinary differential equation, yielding a new numerical method for it.