Faking Brownian motion with continuous Markov martingales

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Abstract: Hamza and Klebaner posed the problem of constructing martingales with Brownian marginals that differ from Brownian motion, so-called fake Brownian motions. Besides its theoretical appeal, the problem represents the guintessential version of the ubiguitous fitting problem in mathematical finance, where the task is to construct martingales which satisfy marginal constraints imposed by market data. In the past 20 years, numerous mathematicians have constructed solutions to this challenge which are either non-continuous (and strongly Markovian) or continuous (but non-Markovian). In contrast, it is known from Gyöngy, Dupire, and ultimately Lowther that Brownian motion is the unique continuous strong Markov martingale with Brownian marginals. We took this as a challenge to construct examples of a "very fake" Brownian motion, that is, continuous Markov martingales with Brownian marginals that miss out only on the strong Markov property.