The intersection of two real forms in a K\"ahler C-space

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Abstract: An orbit of the adjoint representation of a compact connected semisimple Lie group G admits a G-invariant K\"ahler structure, and it is called a complex flag manifold. On the other hand, any K\"ahler C-space, that is, a simply-connected compact homogeneous K\"ahler manifold, can be realized as a complex flag manifold. In this talk, we give the notion of antipodal set of a complex flag manifold. We show that a maximal antipodal set of a complex flag manifold is given as an orbit of a Weyl group of G.
In a K\"ahler manifold, a connected component of the fixed point set of an anti-holomorphic involutive isometry is called a real form. Hence a real

form is a totally geodesic Lagrangian submanifold. We give a necessary and sufficient condition for two real forms, which are not necessarily congruent, in a complex flag manifold to intersect transversally in terms of the symmetric triad. Then we show that the intersection of two real forms is antipodal. As an application, we calculate the $\mathbb{Z}_2$-Lagrangian Floer homology of two real forms in a K\"ahler-Einstein C-space. The Floer homology is generated by the intersection of two real forms.

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