Philipp Schmitt
University of Wuerzburg

Construction of a Locally Convex Topology with respect to which Karabegov's Star Product on Spheres is continuous

Abstract: The idea of deformation quantization is to deform a commutative algebra of observables into a non-commutative one. Usually, deformation quantization refers to formal deformations, i.e. it gives the product in terms of power series of a formal parameter. In physical applications the formal parameter needs to be substituted with a real number, the specific value of Planck's constant, leading to the question whether the formal series converge for a suitable subalgebra of observables.

Sometimes it is easy to write down such a subalgebra, e.g. a subalgebra for which the formal power series have only a finite number of non-zero terms. These subalgebras are usually rather „small". Hence one would like to take their completion with respect to a topology for which the star product is continuous, since, in this case, the star product extends uniquely to the completion. The construction of such a topology for a star product on spheres will be the topic of this talk.

First, I want to recall a construction of star products on coadjoint orbits, which is due to Karabegov. This construction works for all coadjoint orbits of compact, semi-simple, connected and simply-connected Lie groups. Fixing a complex structure on the coadjoint orbit, we can always construct a star product of Wick type. The star product of two fixed functions is a rational function of the deformation parameter $\hbar$, that may have a finite number of poles.

I want to treat the particular case of coadjoint orbits for $SU(2)$, which are spheres. In the case of a unit sphere, poles can only occur for $\hbar = \frac{1}{n}$ with $n \in \mathbb{N}$. I will introduce a locally convex topology, called the T_R topology, on $\mathrm{Pol}(\mathfrak{su}_2)$ and construct a topology on the polynomials on the coadjoint orbit by taking a quotient. With explicit formulas, that express Karabegov's star product as a deformation of the Gutt star product on the Lie algebra $\mathfrak{su}_2$, one can prove that Karabegov's star product is continuous with respect to this quotient topology for $\hbar \neq \frac{1}{n}$.

If time permits, I will say a few words on generalizing this result to coadjoint orbits of arbitrary compact, semi-simple, connected and simply-connected Lie groups.