FINITE GROUPS AND LIE RINGS ADMITTING A FROBENIUS GROUP OF AUTOMORPHISMS WITH FIXED-POINT-FREE KERNEL

E. I. KHUKHRO

Suppose that a finite group G admits a Frobenius group of automorphisms FH with kernel F and complement H such that $C_G(F) = 1$. Then G is soluble by a theorem of V. V. Belyaev and B. Hartley (using the classification of finite simple groups). By Clifford's theorem, every FH-invariant abelian section V of G is a direct sum of $|H|$ subgroups freely permuted by H, so that $C_V(H)$ is the "diagonal", and V is "$|H|$ times $C_V(H)$". Hence it is natural to expect many parameters of G to be close to the same parameters of $C_G(H)$ (possibly, depending on $|H|$). We discuss several recent results of the author, N. Yu. Makarenko and P. Shumyatsky in this direction. Many of the results rely on Lie ring methods, and analogous results are proved for Lie rings admitting Frobenius groups of automorphisms with fixed-point-free kernel.

It is proved that the order and the sectional rank of G are bounded in terms of $|H|$ and the same parameters of $C_G(H)$. It is proved that the Fitting height of G is equal to the Fitting height of $C_G(H)$. The proofs of these results are based on Clifford's theorem.

Lie ring methods are used to prove that if FH is metacyclic and $C_G(H)$ is nilpotent, then the nilpotency class of G is bounded in terms of H and the class of $C_G(H)$. (Earlier the special case of double Frobenius groups was settled by N. Yu. Makarenko and P. Shumyatsky, which gave an affirmative solution of V. D. Mazurov's problem 17.72(a) in Kourovka Notebook.) A similar result is also proved for Lie rings admitting metacyclic Frobenius groups of automorphisms with fixed-point-free kernel. Examples show that in these results the condition of FH being metacyclic is essential. It is conjectured that the dependence on $|H|$ here is essential; but so far it is only known that the nilpotency class of G can be greater than that of $C_G(H)$.

A different Lie ring method is used to prove that if FH is metacyclic, then the exponent of G is bounded in terms of $|FH|$ and the exponent of $C_G(H)$. It is conjectured that here the metacyclicity condition can be dropped. It is also unclear whether the dependence on $|F|$ or $|H|$ is essential; probably, the dependence on $|F|$ can be dropped. There are only examples with exponent of G being greater than that of $C_G(H)$.

There are some other recent results in this area, and there still remain many open problems.

Sobolev Institute of Mathematics, Novosibirsk (Russia)
E-mail address: khukhro@yahoo.co.uk