Complex conference matrices

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Abstract

A complex conference matrix is a square matrix of order n with zero diagonal and unimodular complex numbers elsewhere such that $C^*C = (n-1)I$. Paley used finite fields with odd orders $q = p^m$, p prime and the Legendre symbol to construct real symmetric conference matrices of orders $q+1$ whenever $q \equiv 1 (\mathrm{mod}\, 4)$ and real skew-symmetric conference matrices of orders $q+1$ whenever $q \equiv -1 (\mathrm{mod}\, 4)$. We prove that Paley construction can be extended to the complex setting. Let $\mathbb{F}_q$ be the Galois field of order $q = p^m$, p a prime number and m a positive integer. It is seen in this talk that for any nontrivial multiplicative character $\varkappa$ of $\mathbb{F}_q^*$ and for any $b \in \mathbb{F}_q^*$ we have $\sum_{a \in \mathbb{F}_q^*} \varkappa(a)\overline{\varkappa(a+b)} = -1$. Whenever q is odd and $\varkappa$ is the Legendre symbol this formula reduces to the well-known Jacobsthal's formula. We then use the extended formula to produce a complex symmetric conference matrix of order $q+1$ whenever $q \geq 4$ is any prime power as well as a complex skew-symmetric conference matrix of order $q+1$ whenever q is any odd prime power. These matrices were constructed very recently in connection with harmonic Grassmannian codes, by use of finite fields and the character table of their additive characters. We propose here a new and simple proof of their construction by use of the above generalized formula similarly as was done by Paley in the real case. We also classify, up to equivalence, the complex conference matrices constructed with some nontrivial characters. In particular, it turns out that the complex conference matrix constructed with any nontrivial multiplicative character $\varkappa$ and that one constructed with $\varkappa^{p^k}$ for any integer $k = 1, ...m-1$ are permutation equivalent. We end our talk by some open problems.

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