A Newton method with adaptive finite elements for solving phase-change problems with natural convection

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Abstract

We present a new numerical system using finite elements with mesh adaptivity for the simulation of solid-liquid phase change systems. In the liquid phase, the natural convection flow is simulated by solving the incompressible Navier-Stokes equations with Boussinesq approximation. A variable viscosity model allows the velocity to progressively vanish in the solid phase, through a intermediate mushy region. The phase change is modelled by introducing an implicit enthalpy source term in the heat equation. The final system of equations describing the liquid- solid system by a single domain approach is solved using a Newton iterative algo- rithm. The space discretization is based on a P2-P1 Taylor-Hood finite elements and mesh adaptivity by metric control is used to accurately track the solid-liquid interface.

The numerical method is validated against classical benchmarks that progres- sively add strong non-linearities in the system of equations: (i) natural convection air flow (described by the Navier-Stokes equations with linear buoyancy term), (ii) natural convection of water (introducing nonlinear buoyancy term), (iii) melting of a phase-change material (introducing variable viscosity and nonlinear en- thalphy source term), (iv) water freezing (displaying together all previous non- linearities). Very good agreement with experimental data is obtained for each test case, proving the capability of the method to deal with both melting and so-

lidification problems with convection. The mesh adaptivity method proved very effective in accurately tracking not only liquid-solid interface, but also the water maximum density line in the difficult case of water freezing. The presented numerical method is very easy to implement using FreeFem++ software using a syntax close to the mathematical formulation.